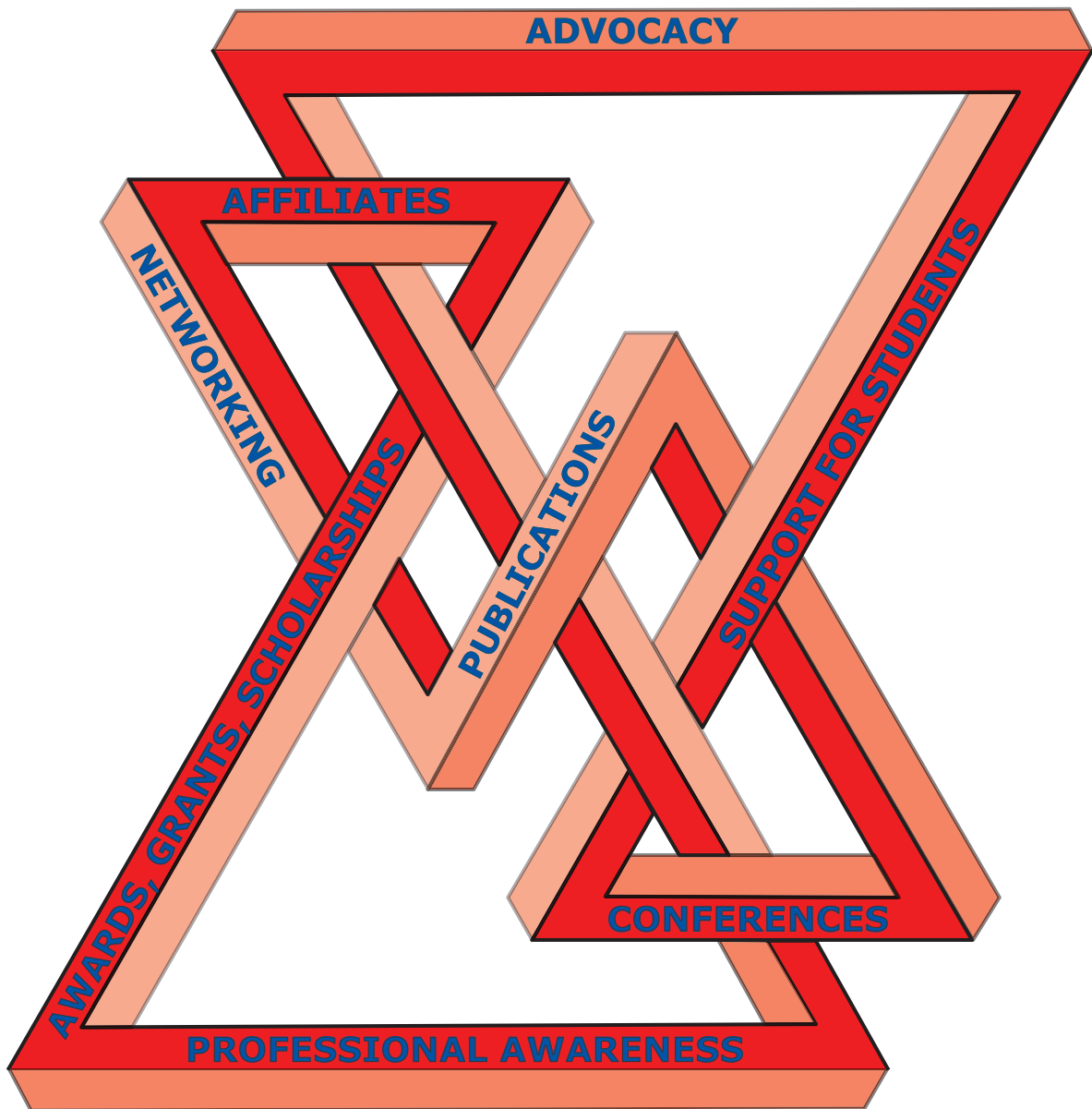


The California Mathematics Council

# ComMuniCator

## Special Membership Issue



An Affiliate of  
The National Council of Teachers of Mathematics  
The National Council of Supervisors of Mathematics

## CMC ComMuniCator Membership Issue

This special membership issue of the *ComMuniCator* is designed to encourage teachers and others interested in mathematics education to become members of the California Mathematics Council (CMC). While the *ComMuniCator* is but one benefit of membership in CMC, we hope that through the information, articles, and activities presented here many will think about becoming members and sharing in the support that teachers of mathematics at all levels, kindergarten through college, receive as CMC members.

Our hope is to distribute this membership issue of the *ComMuniCator* far and wide. If you are a teacher educator, administrator, coordinator, professor, or supervisor

of preservice or inservice teacher programs, please request copies of this *ComMuniCator* for your teachers.

Copies of the membership issue of the *ComMuniCator* are available without charge. If you would like to receive copies, please contact:

Gretchen Muller  
CMC Membership  
PO Box 234  
Kentfield, CA 94914.

For CMC membership information, call 1-888-CMC-Math or go to the CMC website at [www.cmc-math.org](http://www.cmc-math.org).

---

### • CMC MEMBERSHIP FORM •

You can sign up or renew your membership in several ways, depending on the method of payment:

- ✓ online at [cmc-math.org/cmc-membership/member-infojoinrenewal/](http://cmc-math.org/cmc-membership/member-infojoinrenewal/) with a credit card;
- ✓ by phone/fax to **888-CMC-MATH (888-262-6284)** with a credit card;
- ✓ by filling out and mailing the following to **CMC • PO Box 234 • Kentfield CA 94914** with check, credit card, or purchase order.

Name \_\_\_\_\_

Address \_\_\_\_\_

City \_\_\_\_\_ State \_\_\_\_\_ ZIP \_\_\_\_\_

Phone \_\_\_\_\_ E-mail \_\_\_\_\_

District \_\_\_\_\_ School \_\_\_\_\_

Position \_\_\_\_\_ Level(s) of Interest \_\_\_\_\_

**Member Rates** (as of January 2015; go to [www.cmc-math.org/cmc-membership/member-infojoinrenewal/](http://www.cmc-math.org/cmc-membership/member-infojoinrenewal/) for the current membership rates):

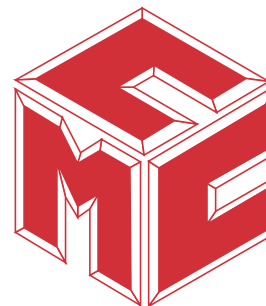
\_\_\_ Regular, \$50 (one year)    \_\_\_ Regular, \$95 (two year)    \_\_\_ Student/Retired, \$25 (one year)  
\_\_\_ Institutional, \$100 (one year)    \_\_\_ Electronic, \$45 (one year)

**Life Membership Rates** (as of October 2014; go to [www.cmc-math.org/cmc-membership/member-infojoinrenewal/](http://www.cmc-math.org/cmc-membership/member-infojoinrenewal/) for the current membership rates):

\_\_\_ \$700 (age 40 & under)    \_\_\_ \$600 (age 41 to 55)    \_\_\_ \$400 (age 56 & over)

# Welcome to CMC

*by the CMC Board*



**T**he California Mathematics Council (CMC) is an organization that serves mathematics educators at all levels—teachers, parents, students, paraprofessionals, student teachers, administrators, and college and university staff.

---

*CMC is an association of people who appreciate the beauty and fascination of mathematics and enjoy sharing that wonder with students of all ages.*

---

As you read through this mini-edition of the *ComMuniCator*, the journal of the California Mathematics Council, keep in mind the many ways that CMC might serve you.

---

## Advocacy . . .

Through its work at the state level, CMC creates opportunities for its members' voices to be heard on issues critical to the teaching of mathematics: a standards-revision process, ongoing assessment of the Common Core Standards, policy changes to better serve teachers and students, and a broader professional development focus for K–12 teachers in the areas of mathematics pedagogy, instructional strategies, content, and instructional materials.

---

*Through CMC, members have a large role in the decision-making process as our state crafts the plan for rigorous and relevant mathematics instruction.*

---

Bigger issues are on the horizon, such as implementation of Common Core and STEM (Science, Technology, Engineering and Mathematics), as CMC strives to represent your views in work with the California Department of Education.

---

## Professional Awareness . . .

CMC keeps its members informed of actions that directly affect the teaching and learning of mathematics, such as *California Mathematics Framework* revisions, development of long-term mathematics policies, implementation of new national standards, and changes in assessment at the state and national levels.

---

*CMC has direct communication with the California State Department of Education and representation in Sacramento.*

---

As a CMC member, you are represented in state committees, task forces, and work groups that are considering issues related to mathematics education in California.

---

## School and Community Support . . . The Math Festival Program [www.cmc-math.org/activities/math\\_festival.html](http://www.cmc-math.org/activities/math_festival.html)

The CMC Math Festival Program is a way for schools to celebrate mathematics.

---

*K–8 students can enjoy a multitude of challenging problem-solving activities during the day and bring their families to a Family Mathematics Festival that evening.*

---

A Math Festival is an exciting school-wide event that exposes students, teachers, and parents to key mathematics topics in a positive,

*continued on page 4 > >*

self-explanatory, festival-like atmosphere.

---

## **Conferences. . .**

CMC organizes and sponsors local and regional conferences that allow concerned individuals to keep current in mathematics education.

---

*CMC offers conferences of national renown in the fall and spring of each year.*

*Northern Section*  
*ASILOMAR*

*Central Section*  
*VARIOUS LOCATIONS*

*Southern Section*  
*PALM SPRINGS*

---

Inservice sessions feature current techniques and materials. Members of CMC receive special considerations, such as early program mailing.

---

## **Awards, Grants, Scholarships. . .**

CMC recognizes people who have made outstanding contributions to the teaching of mathematics.

### *George Polya Award*

Presented yearly to one or more members who are classroom teachers of mathematics and who have been visible as leaders in a state-wide context.

---

### *Edward Begle Award*

Presented yearly to one or more California mathematics educational leaders who are actively supportive of CMC's goals.

---

### *Walter Denham Award*

Presented to a person who regularly

advocates for excellence in mathematics education at the state and/or national level. This Award is for an individual who champions excellence in mathematics teaching and learning by engaging policy makers in rational, respectful discourse supported by data while remaining steadfast in the face of adverse circumstances.

---

### *Presidential Awards*

One award each year to a teacher of mathematics—secondary teachers in odd years and elementary teachers in even years. The process is managed by CMC under the auspices of the National Science Foundation (NSF). Each winner receives a trip to Washington, D.C. and cash awards from the NSF and CMC.

---

### *Lurie Center Scholarship*

Awarded yearly to teachers of color to attend each of the CMC Conferences.

---

### *CMC Section Grants and Awards*

Each of the CMC sections—North, South, and Central—offers grants and awards for preservice and current teachers who reside in their geographic areas.

---

To read more about these opportunities and to access other mathematics education resources, go to the CMC website: [www.cmc-math.org](http://www.cmc-math.org).

---

## **CMC's Affiliates. . .**

CMC has local affiliates throughout the state that offer a wonderful variety of mathematics activities for both teachers and students during the school year. We have local county-wide and regional affiliates plus a few state-wide interest groups.

---

*Visit the CMC webpage for updates to the activities, dues, and links for these organizations. You will find the affiliates listed under each section in the Organization link.*

---

Get involved and extend your influence in improving mathematics education beyond your immediate classroom, school, and district by joining one or more of the CMC affiliates. You can include membership in an affiliate when you renew or join CMC via the online membership link.

---

### **Support for Students. . .**

CMC promotes student involvement and interest in the field of mathematics through student mathematics activities and recognition of outstanding students in mathematics.

---

#### ***Student Activities Trust Fund***

The Student Activities Trust Fund (SATF) helps qualifying applicants sponsor activities directly dealing with students, such as mathematics field days, problem-solving contests, and mathematics olympiads.

---

#### ***Recognition of Outstanding Students***

A certificate signed by the CMC President and the State Superintendent of Schools is provided to each high school in the state to award to an outstanding mathematics student.

---

### **Publications. . .**

CMC publishes information of interest to all individuals involved in mathematics education.

---

***The ComMuniCator, our professional journal, is published quarterly and features articles on current topics of interest to teachers and new activities that are ready for use with students.***

---

Look to the *ComMuniCator* for teaching ideas, resources, and notices of professional meetings, seminars, and conferences.

---

### **CMC on the Internet. . .**

#### ***www.cmc-math.org***

The CMC website is updated regularly and helps our members stay current on important events affecting all our classrooms and students—public and private. Find out and register for our annual conferences, see a calendar of mathematics education events offered by local affiliates and partner organizations, learn about newsworthy mathematics issues in California and across the nation, and access the members-only section to view the *ComMuniCator* online.

---

***CMC is on the Web and on Facebook and Twitter, in order to keep its members informed and involved through the use of social-networking media.***

---

You can post questions, announcements, or contribute to the collective knowledge of our CMC Facebook group. Visit the website to find out more about the projects and activities CMC is involved with and to get a complete list of the CMC leadership.

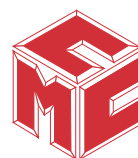
---

### **Networking. . .**

The strength of any organization lies in its membership. If *you* join CMC, we can all do more. CMC requests your participation in our organization and its activities. CMC needs your membership to develop and maintain an effective state organization.

---

***CMC works!  
Let CMC work for you.  
Join the California Mathematics  
Council and get connected!***



## Equity and the Common Core State Standards

by Kathlan Latimer, CMC State Past President

**F**oundational to the Common Core State Standards for Mathematics (CCSSM) is a belief in equity. Some of the goals frequently attributed to the CCSSM initiative include: Every student is ready for career, college, and life. Every student has access to rigorous standards and high expectations. Every student is a graduate.

In its mission statement, the California Mathematics Council espouses a belief in equity: "All students have the capacity to become mathematically competent and confident when provided a rigorous and challenging mathematical program supported by high expectations."

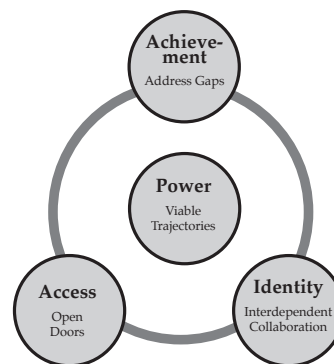
The National Council of Teachers of Mathematics (NCTM) also promotes an emphasis on equity. In its position statement on equity, NCTM (2008) states:

Excellence in mathematics education rests on equity—high expectations, respect, understanding, and strong support for all students. Policies, practices, attitudes, and beliefs related to mathematics teaching and learning must be assessed continually to ensure that all students have equal access to the resources with the greatest potential to promote learning.

We do mean *all* students, regardless of their zip code, first language, skin color, level of income, and any other factors that should not—but often do—serve as screens, obstacles, or barriers.

We support equity in mathematics and now have standards that demand the same for all students in the United States. Many schools and districts have already begun to implement the CCSSM's Standards for Mathematical Practice. As the transition to the full contingent of standards—both practices and content—occurs, we can stand on the bedrock of equity to make sure we have structures, practices, and policies in place to support them.

The Organisation for Economic Co-operation and Development (OECD) has studied equity globally and asserts in its policy brief "No More Failures: Ten Steps to Equity in



Education" (2008) that equity in education has two dimensions: fairness and inclusion. Fairness refers to ensuring that personal or social factors such as language and ethnicity do not bar participation while inclusion means setting a bar that all must reach. That bar is readiness for college and/or career. Through implementation of the CCSSM, fair and inclusive education is sought for all our nation's children. It is important to implement CCSSM within a framework of equitable practices

But how do we accomplish this? What are the strategies, practices, or policies necessary to realize this goal? What is the work? This article will focus on two that appear time and time again in the literature that we can get started working on now: collaboration and involvement.

### Collaboration

The CCSSM provides a platform for states, districts, and classrooms to engage in discussion and to share resources. With the many advances in technology, distance is no longer a barrier and collaboration may now occur in new ways. It is neither necessary nor productive to work in isolation.

Looking at practitioners, *The PRIME Leadership Framework*, created by the National Council of Supervisors of Mathematics (NCSM), details a vision for equity work. Equity Indicator 3 speaks to collaboration, a move from private practice to collaboration: "Every teacher works interdependently in a collaborative learning community to erase inequities in student learning" (2008, p. 18).

This collaboration goes beyond common planning to developing communities of practice which look at best practices, formative assessment, differentiation strategies, analysis of cognitive demands of tasks, early interven-

tion, responding to diversity, and accommodating differences. These communities ultimately answer the question: What is needed to move our students forward? They share ideas and share the work.

A focus on equity means a focus on continuous achievement and the aforementioned foci are central to meeting the varying needs of students. NCTM in its "Tips on Supporting All Students" provides some starting points for discussion. We can start by asking: How do we . . .

- ✓ focus on the individual student?
- ✓ create an environment for success?
- ✓ identify (and get over) biases?
- ✓ create an equitable curriculum that supports diverse needs and celebrates diverse strengths?
- ✓ become aware of questioning and listening techniques that support all students?

Teachers need the means and opportunities to do this work. How will it occur: professional learning communities, online groups, grade/department meetings, . . .? We need to find ways to carve out time for this important work now.

## Involvement

Implementation of CCSSM should not occur in a vacuum. All stakeholders have a role to play and community support is needed. We can start with information on the shifts that will occur and the resources needed for success. We can build partnerships that require not only ongoing relationships and input on implementation, but also responsibility for advocacy and identifying and obtaining needed resources. Returning to NCTM's position statement on equity, stakeholders become a part of the continual assessment process. It becomes everyone's job to make sure that our students achieve.

Within the community we must look to a specific group—parents. The CCSSM lets parents know what is expected of their children in order to provide support and assistance. In its policy brief, the OECD looks at equity from a global perspective and provides steps towards equity in areas of design, practice, and resourcing. One practice that is not new to us is step 6: "Strengthen the links between school and home to help disadvantaged parents

help their children to learn" (p. 6). We must help all parents, but especially those who are disadvantaged or marginalized, know of the changes to come and the roles they can play to assist. We must help parents help their children and we must strengthen ties between home and school.

What will it take to bring more families on board, particularly those now reluctant to participate in their children's schooling and school community? Perhaps parent centers, increased efforts to communicate, or language support? We need to create a purposeful plan for inclusion of parents in the school community and this may mean leaving the schoolhouse and going into the community. We need everyone.

## All to say. . .

We can prepare now for full implementation; many are already implementing the CCSSM's Standards for Mathematical Practice. Beyond work on these standards, we can also work to build structures that support equity or bolster those already in place, and take careful steps to ensure that our systems and classrooms are fair and inclusive and that stakeholders are on board so that all of our students are indeed ready for the life that awaits them.

There is much to do. Let us hit the ground running.

## References

- National Council of Supervisors of Mathematics (NCSM). 2008. *The PRIME Leadership Framework: Principles and Indicators for Mathematics Leaders*. Bloomington, IN: Solution Press.
- National Council of Teachers of Mathematics (NCTM). 2008. "Equity in Mathematics Education." <<http://www.nctm.org/about/content.aspx?id=13490>>.
- \_\_\_\_\_. "Tips on Supporting All Students: Equity and Diversity." <<http://www.nctm.org/resources/content.aspx?id=15863>>.
- Organisation for Economic Co-operation and Development. 2008. "No More Failures: Ten Steps to Equity in Education." <<http://www.oecd.org/dataoecd/49/16/b49623744.pdf>>.

*For updated resources and information on the California Common Core State Standards for Mathematics, refer to the CMC web site at [www.cmc-math.org/](http://www.cmc-math.org/).*

# Algebra and Patterning for Kindergarten/ First Grade Students

by Fran Threewit, Kenwood Elementary



**CONCEPTS:** Counting & Cardinality, Operations & Algebraic Thinking, Number & Operations in Base Ten

**SKILLS:** Identifying, describing, and extending simple patterns; collecting data and recording results; using operational symbols

**MATHEMATICS STANDARDS:** K.CC.2, K.OA.1, K.OA.2, K.MD.3; 1.OA.1, 1.OA.8, 1.MD.4

**MATHEMATICS PRACTICES:** 1, 2, 3, 5, 7

**GRADES:** K–1

**MATERIALS:** Classroom objects, manipulatives (pattern blocks, linking cubes, beads, etc.), stickers, rhythm instruments, pocket chart, classroom calendar

## BACKGROUND

It is important for kindergarten/first grade students to use a variety of concrete materials and hands-on experiences as they embark on the early study of algebra. They should begin the process of learning about variables by using pictures and symbols to look for and describe patterns. While they work toward relating to addition and subtraction they can look for unknown numbers and they can write number sentences about real-world situations.

## DESCRIPTION

Listed below are some activities that illustrate how algebraic concepts can be introduced and integrated in the K/1 classroom.

- ◆ Sort and pattern objects by color, size, shape, and kind. Use objects found in the classroom, such as crayons, pens, pencils, chairs, etc.
- ◆ Identify patterns in the environment, in pictures, and in lines of objects, and explain how the items are arranged. Use linking cubes, beads, macaroni, pattern blocks.
- ◆ Create and describe a pattern of objects or numbers. Extend the pattern up to three or more times. Use pattern strips made from stickers or stamps and stamp pads for students to continue the pattern.

- ◆ Create rhythmic patterns with claps, taps, or on a rhythm instrument. Echo a short invented rhythmic pattern played by another student.
- ◆ Relate patterns to number.
  - Use the pocket chart to sequence number cards on a number line. Gradually replace the number line with number strips that follow a pattern or have a missing number in the pattern sequence.
  - Use the classroom calendar to create numerical patterns with pictures and numbers. Count by 1s, 2s, 5s, and 10s. Find odd and even numbers.
  - Connect unknown numbers by drawing pictures and writing number sentences that describe a pattern of pattern blocks, linking cubes, or beads.

## CONCLUSION

There are many websites that will extend already introduced patterning or numerical lessons. Many such websites are available by doing a search for “kindergarten + mathematics + patterning.” A few examples are:

[www.primarygames.com/patterns/start.htm](http://www.primarygames.com/patterns/start.htm)

[www.bbc.co.uk/education/laac/music/fdl.html](http://www.bbc.co.uk/education/laac/music/fdl.html)

[mathforum.org/varnelle/patterns.html/](http://mathforum.org/varnelle/patterns.html/)

With consistent experiences and activities and through the use of concrete materials, students can develop a beginning understanding of algebraic concepts. Not only will students enjoy each of the described activities, they will also be introduced to new understandings.



# Number Chart Puzzles

by Scott Farrell, Conejo Valley USD

**CONCEPTS:** Operations & Algebraic Thinking, Numbers & Operations in Base Ten

**SKILLS:** Understanding place value, counting, applying number patterns

**MATHEMATICS STANDARDS:** 1.OA.3, 1.OA.4, 1.NBT.1, 1.NBT.4, 1.NBT.5; 2.OA.2, 2.NBT.2, 2.NBT.5; 3.NBT.2

**MATHEMATICS PRACTICES:** 1, 3, 6, 7, 8

**GRADES:** 1–3

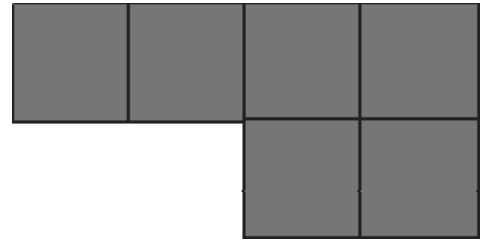
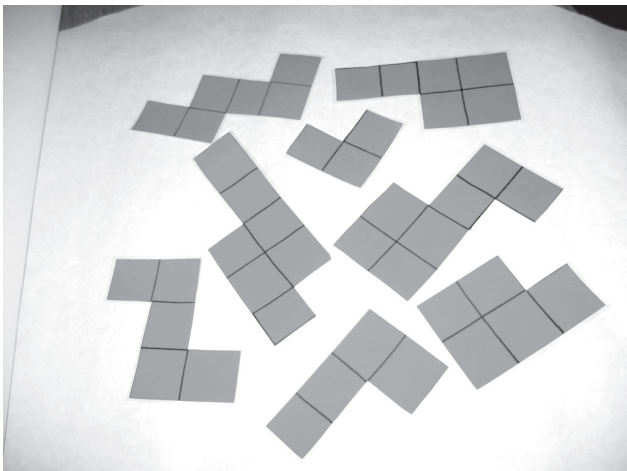
**MATERIALS:** Large hundreds chart, puzzle pieces, dry erase pens

## BACKGROUND

This activity provides practice in reconstructing number charts using number patterns. Before playing the game children need to be familiar with a hundreds chart and have had several opportunities to record to 100 on a blank number chart. Also whole group activities where students are required to identify missing numbers by covering them with a puzzle piece are beneficial.

## DIRECTIONS

Before the lesson the teacher makes several puzzle pieces from laminated construction paper. Each piece needs to be a different shaped puzzle piece that covers five to eight number spaces on the large hundreds chart. The pieces are reused during the game since they are erased and flipped and placed in different locations on the number chart.



## Playing the Game

1. The class is divided into two equal teams.
2. Each team will select one member to choose a puzzle piece to place anywhere on the large number chart. Fasten the puzzle piece using tape or magnets.
3. The opposing team will pick one member to write in the missing numbers using a dry erase pen.
4. The teams will then switch roles and continue so that all students have the opportunity to fill in a puzzle piece.

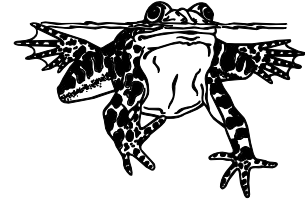


## VARIATIONS

- ◆ Larger numbers such as 101 to 200 can be used on the number charts.
- ◆ Students can play with a partner using individual number charts and creating their own puzzle pieces from grid paper.

# What If You Hopped Like a Frog?

by Patty Montgomery, Ditmar Elementary School, Oceanside USD



**CONCEPTS:** Operations & Algebraic Thinking, Number & Operations in Base Ten, Number & Operations: Fractions, Measurement, Ratios & Proportional Relationships

**SKILLS:** Multiplying and dividing, measuring weights and heights, finding proportional relationships

**MATHEMATICS STANDARDS:** 4.OA.1, 4.OA.2, 4.NF.4a, 4.MD.1, 4.MD.2; 5.NBT.5, 5.NF.4, 5.NF.5, 5.MD.1; 6.RP.3; 7.RP.1

**MATHEMATICS PRACTICES:** 1, 2, 3, 4, 5, 6  
**GRADES:** 4–8

**MATERIALS:** *If You Hopped Like a Frog* by David Schwartz, measuring tapes, bathroom scale, copies of the problems (Activity Sheet, page 11)

## BACKGROUND

Author David Schwartz uses proportional reasoning to determine what it would be like if we had the abilities or dimensions of familiar animals. For example: “If you hopped like a frog, you could jump from home plate to first base in one mighty leap.” Artist James Warhola easily captures the attention and imagination of fourth and fifth grade students. After twelve of these fascinating proportions, Schwartz provides some factual data on which the proportions are based.

Many students at this age are still very literal and can be confused by the proportional reasoning aspect. When I read this book to my class, my students were all “a-buzz” about each possibility presented in the book. After the page, “If you were as strong as an ant. . . you could lift a car!” I heard a child comment, “An ant can’t do that!”

## PREPARATION

Copy one set of problems for each student. Attach measuring tapes to the walls in a few different locations to cut down on waiting time.

## DIRECTIONS

Read the book to the class and show the pictures as you read the story. Distribute the problems. After reading a few of the problems, ask the students what information they would need to know about themselves to solve these

problems. They will quickly realize that they need to know their height and weight.

Since height and weight can be sensitive issues for some children, allow them to work alone and keep their answers to themselves. Some students already know their height and weight so they may use those numbers.

Since problem five doesn’t require any measuring, I encouraged students to begin with it while I assisted other students with finding their weight. Give all students the opportunity to find their weight in private. I had one table group at a time come up, but kept a distance between the person getting weighed and the rest of the students. Since most had never used this type of scale, I assisted the children individually and helped them read their weights privately.

Allow students to choose to use metric or U.S. customary units when finding their heights. My students realized that using metric units would give them larger numbers. I was pleased that some students chose to challenge themselves by using the larger numbers.

After students know their height and weight, discuss the advantages of rounding their numbers and why rounding is close enough for this activity.

Tell students to solve the problems any way they can, showing all their work. My students had a great time figuring out what they would be able to do if they were built like these animals.

## EXTENSIONS

- ◆ Using answers from problem number one, ask students to determine where they would land if they hopped out the classroom door.
- ◆ Using answers from other problems, have students convert answers to other units of measurement.

## Reference

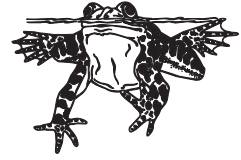
Schwartz, David. 1999. *If You Hopped Like a Frog*. New York: Scholastic.

Activity Sheet, page 11 . . .

## If You Hopped Like a Frog

by Patty Montgomery

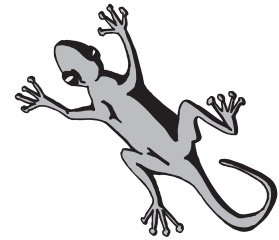
1. A 3-inch frog can hop 60 inches. That means the frog is jumping **20 times** its body's length. How tall are you? \_\_\_\_\_ If you could jump 20 times your body length, how far could you go? \_\_\_\_\_



2. A shrew that weighs  $\frac{1}{5}$  of an ounce eats about  $\frac{3}{5}$  of an ounce of insects and worms each day. That means the shrew eats **3 times** its own weight daily! How much do you weigh? \_\_\_\_\_ How many pounds of food would you eat if you ate 3 times your weight? \_\_\_\_\_



3. A 1-foot chameleon may have a 6-inch tongue. Its tongue is **half as long** as its body. How tall (long) are you? \_\_\_\_\_ How long would your tongue be if you had a tongue like a chameleon's? \_\_\_\_\_



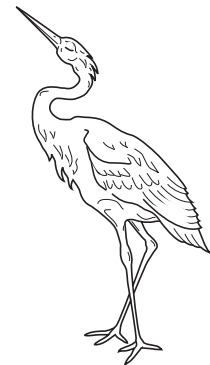
4. An ant weighing  $\frac{1}{250}$  of an ounce can easily lift a breadcrumb weighing  $\frac{1}{5}$  of an ounce. That means the ant is lifting **50 times** its own weight. How much do you weigh? \_\_\_\_\_ How much weight could you lift if you could lift like an ant? \_\_\_\_\_



5. An eagle sees about **5 times** as well as you do. From the air, you can spot a rabbit about 300 meters away. With eagle eyes, you could spot it 5 times as far as that. How far away would you be able to see if you could see like an eagle? \_\_\_\_\_

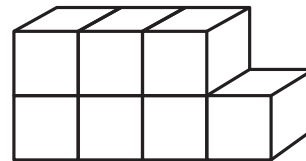


6. A whooping crane's neck is about  **$\frac{1}{3}$  of its height**. How tall are you? \_\_\_\_\_ How long would your neck be if it were  $\frac{1}{3}$  of your height? \_\_\_\_\_



# Counting on Cubes

by Beth Schlesinger, San Diego



**CONCEPTS:** Operations & Algebraic Thinking, Measurement & Data, Expressions & Equations, Geometry, Functions  
**SKILLS:** Extending patterns, finding a general rule using variables

**MATHEMATICS STANDARDS:** 4.OA.5, 4.MD.3; 5.OA.1, 5.OA.3, 5.MD.3, 5.MD.4; 6.EE.2, 6.EE.4, 6.EE.6; 7.EE.2, 7.EE.4, 7.G.6; 8.F.1, 8.F.2, 8.G.9; A-SSE.3; A-CED.1; F-IF.2; F-BF.1

**MATHEMATICS PRACTICES:** 1, 2, 3, 4, 5, 8

**GRADES:** 4–9

**MATERIALS:** Paper, pencils, small cubes such as snap cubes, Activity Sheet (page 13)

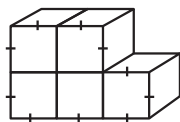
## DESCRIPTION

In this activity students build models out of cubes to represent sequences of perimeters, volume, and surface area. For each sequence on the Activity Sheet, have students work in pairs to build the next three cube models using small cubes such as snap cubes to continue the sequence. Then have them use the cube models to gather data and complete the table, not including the  $n$ th row. Encourage students to verbalize the rule for the  $n$ th term using the previous number, i.e., “Add two to the number that comes before.” Then have them write a rule using the variable  $n$ .

Discovering and formulating these rules is a challenge, but this task is the heart of the activity.

## Some Helpful Suggestions

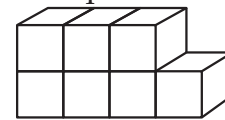
◆ **Counting Perimeter, Volume, and Surface Area.** This model is the second model in Sequence 2 on the Activity Sheet. It has a front face perimeter of ten units and a volume of five cubic units. To find the surface area, you have five square units on the front face, five on the back face, three on the top face, three on the bottom face, and two on each side face for a total surface area of 20 square units.



◆ **Helping Students Formulate and Write Variable Expressions.** You will probably need

to guide students when they are trying to find expressions representing the  $n$ th term. They may come up with algebraic expressions that are valid but not in the simplest form. If so, this is a good opportunity to talk about equivalent expressions.

First, students need to understand what  $n$  represents. The table on the Activity Sheet indicates that  $n$  represents the model number or the term number in the sequence. The model shown here is the third model in Sequence 2. Sometimes by looking at the model students can figure out an expression. For example, to find the volume of this model, for which  $n = 3$ , they might see two rows of three cubes plus one cube and write the expression  $2n + 1$ . Next they should check to see whether this expression works for the other models in the sequence.



To find the total surface area for this piece, they would count seven front squares, seven back squares, two squares on each side, four squares on the top, and four squares on the bottom to get a total of 26 square units. Using variables they might get  $2n + 1$  square units for the front face,  $2n + 1$  for the back face,  $n + 1$  for the tops,  $n + 1$  for the bottom, and 2 for each side for a total of  $6n + 8$  square units.

Another approach, which may not be appropriate for all classes is the following: For Sequence 1, the surface areas form the sequence 6, 14, 22, 30, . . . Start with six and add eight  $(n - 1)$  times. This sequence suggests the general formula  $6 + (n - 1) \times 8 = 6 + 8(n - 1) = 6 + 8n - 8 = 8n - 2$ .

## EXTENSION

Ask students to use the cubes to build their own sequence, make a table, and find expressions to represent the perimeter of the front face, the volume, and the total surface area.

## SOLUTIONS

Set 1:  $4n$ ,  $2n - 1$ ,  $8n - 2$

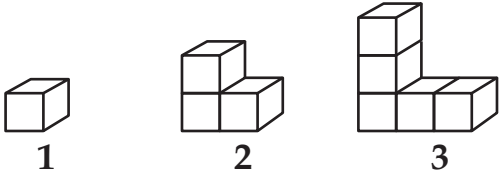
Set 2:  $2n + 6$ ,  $2n + 1$ ,  $6n + 8$

Activity Sheet, page 13 . . .

# Counting On Cubes

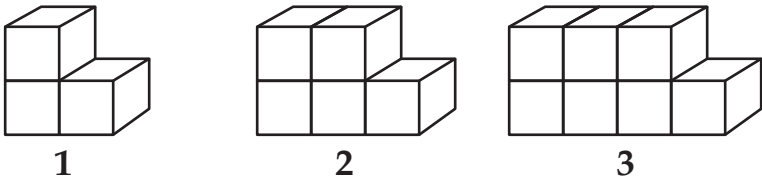
*by Beth Schlesinger*

**Sequence 1**



Model Number	Perimeter of Front Face in Units	Volume in Cubic Units	Total Surface Area in Square Units
<b>1</b>			
<b>2</b>			
<b>3</b>			
<b>4</b>			
<b>5</b>			
<b>6</b>			
...	...	...	...
<i>n</i>			

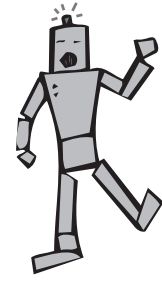
**Sequence 2**



Model Number	Perimeter of Front Face in Units	Volume in Cubic Units	Total Surface Area in Square Units
<b>1</b>			
<b>2</b>			
<b>3</b>			
<b>4</b>			
<b>5</b>			
<b>6</b>			
...	...	...	...
<i>n</i>			

# AlgeBot—A Trek Through the Hundreds Chart

by Brad Fulton and Bill Lombard



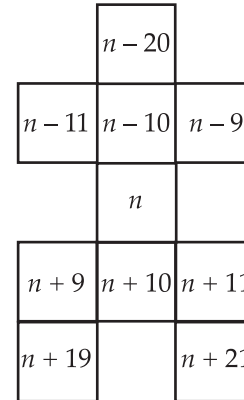
My 8th grade students enjoy discovering the algebraic patterns hidden within the hundreds chart. They are all familiar with this chart that they have seen since Kindergarten. One of our activities involves “AlgeBot.” I display a transparency of the hundreds chart and a second transparency of AlgeBot on top of it as shown.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

I move the upper transparency around until a student yells, “Stop.” Then the students race me to find the total of the numbers inside AlgeBot. The total for the placement shown is 580. I always win this contest and boast that it is because I am so talented at mathematics. The students insist there is a trick and demand a rematch. After a few times, some of the students see a pattern and discover the trick. (Turns out I’m not a rocket scientist after all!) Once a few discover my secret, we explore the problem using algebra.

The students notice that for any number on the chart, the number to the right is one more and the number to the left is one less. Also, the number beneath the given number is ten more and the number above it is ten less. This is always true no matter where AlgeBot runs. If we think of the number in its waist as  $n$ , then the following diagram shows the values of all

ten numbers inside AlgeBot:



If we add the terms inside him, we get  $10n + 20$ . It is a simple matter to look at the number in its waist (56), multiply it by ten (560), and add 20 (580). Many of my students say that they got the answer a different way. Often they simply look two spaces to the right of his waist and put a zero after the number. I express this algebraically on the board. The number two spaces to the right of his waist would be called  $n + 2$ . Putting a zero after it is the same as multiplying by ten. Thus their method is  $10(n + 2)$ . I show them that these are equivalent, being the distributed and factored forms of the same expression:  $10n + 20 = 10(n + 2)$ .

AlgeBot works on any hundreds chart, even one that begins with  $-49$  and ends with 50. It also works on a calendar, but the formula for its sum is slightly different.

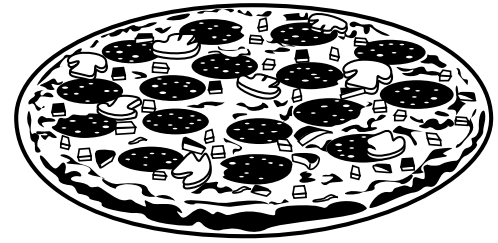
Challenge your students to find the formula when AlgeBot runs around on a calendar. This is just one of many algebraic explorations that can be made on the hundreds chart. If your students are like mine, they will want to explore all the “what ifs” they suggest. One time my students wanted to find out what happens to AlgeBot’s formula when it is standing on its head. Now I was curious.

## Extension

Challenge students to invent new AlgeBots.

# The Pizza Problem

by Milton Rosa, San Juan Unified School District, and Daniel Orey, Universidade Federal de Ouro Preto, Minas Gerais, Brazil



**CONCEPTS:** Number & Quantity, Algebra, Functions, Statistics

**SKILLS:** Using tables, graphs, and regression equations

**MATHEMATICS STANDARDS:** N-Q.1, N-Q.2, N-Q.3; A-SSE.3, A-CED.1, A-REI.10;

F-IF.4, F-IF.7, F-BF.1, F-LE.5; S-ID.1

**MATHEMATICS PRACTICES:** 1, 2, 3, 4, 5, 7, 8

**GRADES:** 9–12

**MATERIALS:** EXCEL, graphing calculators, or graphing applications

## BACKGROUND

Proportional reasoning is the ability to understand and compare ratios. It is also defined as the comparison between two quantities. Proportional reasoning does not depend on specific mechanical skills nor does it depend on algorithmic procedures. Proportional reasoning involves concepts of quantitative and qualitative thinking. This concept allows teachers to develop, elaborate, and present meaningful lessons that show the application of proportional reasoning to students' daily lives

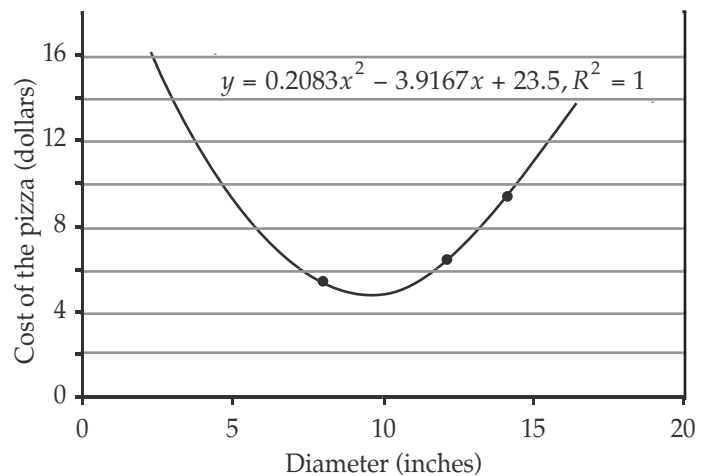
Generally, linear and quadratic equations are often used to model real-world situations that have certain features such as values that depend on surface area. For example, the area  $A$  of a circular region is a quadratic function of the radius  $r$ . The formula for a circular region is  $A = \pi r^2$ . In a particular case the cost of a plain pizza is a function of its area and therefore a quadratic function of its radius or diameter.

## DIRECTIONS

Display the following table on a transparency on an overhead projector or on the board.

Diameter (inches)	2	4	6	8	10	12	14
Cost of Pizza				\$5.50		\$6.50	\$9.50

Suppose that an 8-inch diameter, 12-inch diameter and 14-inch diameter pizza cost, \$5.50, \$6.50, and \$9.50 respectively. Ask your students to use EXCEL, a graphing calculator, or a graphing application to make a graph and find the quadratic equation that best fits the data in the table.



The students should then complete the table with the costs of the pizzas.

Discuss with the students the relationship between the diameter and the cost of the pizza. Pose the following questions:

- ✓ Which pizza has the lowest cost? Is this situation possible? Explain your answer completely.
- ✓ Which pizza has the highest cost? Are these situations possible? Explain your answer completely.

Visit the CMC website  
[www.cmc-math.org](http://www.cmc-math.org)  
 to stay current on important  
 mathematics events affecting all  
 our classrooms and students.



California Mathematics Council  
Phone 888-CMC-MATH  
www.cmc-math.org  
PO Box 234  
Kentfield CA 94914  
CHANGE SERVICE REQUESTED

Non-Profit  
U.S. Postage  
**PAID**  
Sacramento, CA  
Permit #604

## *THE CMC VISION STATEMENT*

### **The California Mathematics Council. . .**

believes that all students have the capacity to become mathematically competent and confident when provided a rigorous and challenging mathematical program supported by high expectations.

### **The California Mathematics Council is committed to:**

- ◆ promoting professional activities that will ensure continual improvement towards excellence in the teaching of mathematics;
- ◆ communicating with educators, parents, the public, and legislative bodies concerning issues related to teaching rigorous, challenging mathematics; and
- ◆ increasing the diversity of the membership of the California Mathematics Council and the diversity of leadership in mathematics education at the local, state, and national levels.