5 Practices for Orchestrating Productive Mathematical Discussions: Step 0

PART 0: SELECTING A MATHEMATICAL TASK	
What are your mathematical goals for the lesson? (I.e., what do you want students to know and understand about mathematics as a result of this lesson?)	There is a one-to-one correspondence between the set of real numbers and the points on a number line. This idea will build from discrete sets, such as whole numbers and integers, to include rational numbers in later grades, and then irrational numbers.

1 Considering the mathematics of the task:
  1 What are your expectations for the mathematics the students will be engaged in as they work on and complete this task?
  2 What Domain/Conceptual Category of mathematics are we targeting?
  3 What content standards will this task target?
  4 What Claims are being targeted with this task?
  5 What Depth of Knowledge Level will the task target?
  6 Which of the Standards for Mathematical Practice are you targeting for students with this task?
	1 **Mathematics:** Representing numbers on the number line and justifying their placement. Using the number system appropriate for a given grade level.
	  • K-2: whole numbers
	  • 3-6: fractions
	  • 7-8: real numbers
	  • High school: real numbers including rational exponents
	2 **Domains:**
	  • Operations and Algebraic Thinking: K, 1
	  • Measurement and Data: 2,5
	  • Number and Operations, Fractions: 3,4
	  • The Number System: 6,7,8
	  • Number and Quantity: High school
	3 **Content Standards** (included but not limited to the following):
	  ◦ **Kindergarten: Operations and Algebraic Thinking K.OA**
	    ▪ Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
	      1 Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
	      2 Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
	  ◦ **Grade 1: Operations and Algebraic Thinking 1.OA**
	    ▪ Represent and solve problems involving addition and subtraction
	      1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
	      2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.
	  ◦ **Grade 2: Measurement and Data**
	    ▪ Relate addition and subtraction to length.

6. Represent whole numbers as lengths from o on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

- o **Grade 3: Number and Operations—Fractions**
  - ▪ Develop understanding of fractions as numbers.
    - 2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
- o **Grade 4: Number and Operations—Fractions**
  - ▪ **Understand decimal notation for fractions, and compare decimal fractions.**
    - 6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*
- o **Grade 5 : Measurement and Data**
  - ▪ Represent and interpret data.
    - 2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *E.g., given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*
    - 6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*
- o **Grade 6: The Number System**
  - o Apply and extend previous understandings of numbers to the system of rational numbers.
    - 6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates
- o **Grade 7: The Number System 7.NS**
  - o Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
    - 1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- o **Grade 8: The Number System 8.NS**
  - ▪ Know that there are numbers that are not rational, and approximate them by rational numbers.
    - 2. Use rational approximations of irrational numbers to compare the the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions
- o **HS: Number and Quantity Overview**
  - ▪ The Real Number System
    - • Extend the properties of exponents to rational exponents

4. **Claims**
- o Primary Claim 3: Communicating Reason: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of other.

5 Practices for Orchestrating Productive Mathematical Discussions: Step 0

	o Secondary Claim 1: Concepts and Procedures: Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.
	5. **Depth of Knowledge Levels**
	  o If enacted at highest level: DOK Level 3: Strategic Reasoning
	  o If enacted at a lower level: DOK Level 2: Skills and Concepts
	6. **Standards for Mathematical Practice**
	  1. Make sense of problems and persevere in solving them.
	  2. Reason abstractly and quantitatively
	  3. Construct viable arguments and critique the reasoning of others.
	  6. Attend to precision
	  7. Look for and make use of structure

Task Criteria	Worthwhile Mathematical Tasks	Score				Comments

After reviewing possible tasks, use the Task Evaluation Form on the right to evaluate the task based on the following rubric:
  1   No evidence of this element in the task, and/or the task does not lend itself to having this element built into it.
  2   This element is included in minor ways, or it appears that incorporating this element is possible.
  3   This element is evident in this task and is important to the success of the lesson.
  4   This element is central to the task or explicit in the design of the lesson.
	The Task…					

	1. Is built around Important, useful mathematics	1	2	3	**4**	The size of numbers and their relationship to each other.
	2. Requires higher-level thinking and problem solving	1	2	3	**4**	Reasoning to determine units between given points on the number line.
	3. Contributes to the conceptual development of students	1	2	3	**4**	Reasoning around the size of numbers and their relationships to each other.
	4. Provides formative assessment opportunities	1	2	3	**4**	Listening to student thinking as they discuss each problem
	And…					
	5. Allows for multiple entry points and solution strategies	1	2	3	**4**	thinking globally or in small increments to determine size of unit
	6. Allows for multiple claims for which evidence can be provided	1	2	3	**4**	Claims based on strategies and reasoning used.
	7. Encourages student engagement and discourse	1	2	3	**4**	Through group approach in which discourse is key
	8. Connects to other important mathematical ideas and/or ideas in other disciplines	1	2	3	**4**	Builds to coordinate plane and then complex numbers
	9. Promotes the skillful use of mathematics	1	2	3	**4**	Through reasoning to determine the size of a unit.
	10. Provides an opportunity to practice important skills	1	2	3	**4**	Deductive Reasoning based on clues given.

5 Practices for Orchestrating Productive Mathematical Discussions: Step 0

	11. Displays sensitivity to and draws on students' diverse background experiences and dispositions	**1**	2	3	4	Task does not lend itself to this element.	
	12. Promotes the development of all students' disposition to do mathematics	1	2	3	4	Through reading, writing, listening and speaking about mathematics.	

Possible Resources for Tasks:

1	Illustrative Mathematics	1	http://www.illustrativemathematics.org/standards/k8 K-8
			http://www.illustrativemathematics.org/standards/hs 9-12
			http://www.illustrativemathematics.org/standards/practice SMP
2	Inside Mathematics	2	http://www.insidemathematics.org/index.php/mathematical-content-standards (k-12)
3	NCTM Illuminations	3	http://illuminations.nctm.org/Lessons.aspx (k-12)
4	Annenberg Learner	4	http://www.learner.org/resources/browse.html?discipline=5&grade=0
5	Thinkfinity	5	http://www.thinkfinity.org/community/thinkfinity-resources (k-12)

SMP	Claims	DOK
1 Make sense of problems and persevere in solving them. 2 Reason abstractly and quantitatively. 3 Construct viable arguments and critique the reasoning of others. 4 Model with mathematics. 5 Use appropriate tools strategically. 6 Attend to precision. 7 Look for and make use of structure. 8 Look for and express regularity in repeated reasoning.	1 Concepts and Procedures: Students can explain & apply mathematical concepts & interpret & carry out mathematical procedures with precision & fluency. 2 Problem Solving: Students can solve a range of complex, well-posed problems in pure & applied mathematics, making productive use of knowledge & problem-solving strategies 3 Communicating Reasoning: Students can clearly & precisely construct viable arguments to support their own reasoning & to critique the reasoning of others. 4 Claim 4: Extended Reasoning: Students can analyze complex, real-world scenarios & can construct & use mathematical models to interpret & solve problems.	1 Recall and Reproduction 2 Skills and Concepts 3 Strategic Reasoning 4 Extended Reasoning